Shape Optimization of Bi-Material Bonded Joint Based on Particle Swarm Algorithm

Mohammad Zehsaz\textsuperscript{a}, Farid Vakili Tahami\textsuperscript{b}, Vahid Khamesi\textsuperscript{c}*

\textsuperscript{a,b,c} Department of Mechanical Engineering, University of Tabriz, Tabriz, Iran

Abstract

In this paper, interface of bi-material has been optimized using ANSYS parametric Design Language (APDL) in order to reduce the stress concentration without stress singularity. In this code, Particle swarm algorithm has been used as an optimization algorithm. The present approach is consisted of two steps: first, the minimum critical angle where stress singularity occurs is determined using characteristic equation; second, geometrical modelling of bi-material interface edge is done using Bezier curve. The results show that with constant Poisson’s ratio, stress concentration decreases by increasing the ratio of elastic modulus. Also, with changing the ratio of elastic modulus, the obtained optimum interface is changed. Numerical examples display the stress concentration reduces significantly in computational optimal curve in comparison to traditional circular shapes.

1. Introduction

Bi-material bonded joints have been in wide use in automotive, aerospace and electronic components applications. Strength and reliability of these joints depend on the bond geometry and material properties of the components. Stress singularity may occur in an elastic solid when discontinuities are present in the geometry, materials or loads, such as cracks, multilayer media or point forces. Due to the extremely high stresses near the points of discontinuity, failure is usually initiated at such locations [1]. In these kinds of materials, failure is mostly created near the corner of bonded location and free surface of each component, demonstrating existence of stress concentration. It is essential to eliminate singular point for improving strength of bi-material bonded joint. Niu and et al. [2] studied the stress singularities in V-notch components in isotropic dissimilar materials. Xu and et al. [3] designed a convex contact surface for measurement of tensile strength. Wu [4] investigated elimination of stress singularity near the apex in a bi-material bonded joint based on an eigenvalue analysis along with finite element approach. He utilized the elimination of the stress singularities to design components. Hideo [5] stated that the stress singularity in three-dimension model is more severe than two-dimension model near corners of bi-material joint. Hu and et al. [6] performed the shape optimization of bi-material single-lap joints. The corresponding shape-optimization approaches were proposed on the basis of sequential linear programming. Furthermore, they studied the influences of material properties, lap length and critical value of intensity of stress singularity on the strength and optimum design shapes. Baladi and et al. [7] investigated effect of joining angle in the interface of aluminum-polycarbonate. Computer simulation and finite element analysis by ABAQUS showed that convex interfacial joint leads to stress reduction at junction corners in comparison with straight joint. Bogy [8] applied the martingale theory to analyze the convergence of the standard PSO (SPSO). Firstly, the swarm state sequence is defined and its Markov properties are examined according to the theory of SPSO. Two closed sets, the optimal particle state set and optimal swarm state set, are then obtained. Afterwards, a super martingale is derived as the evolutionary sequence of particle swarm with the best fitness value. Finally, the SPSO convergence analysis is carried out in terms of the super martingale convergence theorem. Wu [9] proposed a hybrid algorithm combining firefly and particle swarm optimization (HFPSO). The proposed algorithm is able to exploit the strong points of both particle swarm and firefly algorithm mechanisms. Bogy and et al. [10] developed a new approach for structural shape optimization,
which consists in coupling the particle swarm optimization (PSO) algorithm and the isogeometric boundary element method (IGA-BEM). Kennedy and et al. [11] presented the statistical analysis of vortex particle swarm optimization (VPSO) which is a boost algorithm based on self-propelled particle swarms.

This paper studies shape optimization of bimaterial bonded joint based on particle swarm algorithm using ANSYS Parametric Design Language (APDL) which receives the studied parameters as input and obtains the optimum shape for the components studied in this research.

2. Determination of singular stress angle

Firstly, for determination of optimum shape, angles where stress becomes singular should be obtained. Consider the bi-material geometry shown in figure 1. \( \theta_1 \) and \( \theta_2 \) are the angles between interface of a bi-material and free surface of the first and second materials, respectively. \( W \) is the width of geometry.

![Figure 1. Geometry of a bi-material bonded wedge](image)

Both of materials are elastic, isotropic and homogeneous and elastic modulus of the first material is bigger than the second one. The two dimensionless parameters of dissimilar materials with four elastic constants are defined as:

\[
\alpha = \frac{E_1 - E_2}{E_1 + E_2}
\]  

(1)

With

\[
\bar{E}_j = \begin{cases} E_j & \text{in plane stress} \\ \frac{E_j}{1-v_j} & \text{in plane strain} \end{cases}
\]

(2a)

And

\[
\beta = \frac{\mu_1(1 - 2k_1) - \mu_2(1 - 2k_2)}{2[\mu_1(1-k_1) + \mu_2(1-k_2)]}
\]

(2b)

Where, \( E_j \) and \( v_j \) are the elastic modulus and Poisson’s ratios of materials 1 and 2, \( M_j = E_j/(2(1+v_j)) \) is the material shear modulus. The parameter \( \alpha \) is a measure of the relative stiffness of materials 1 and 2, and can take values in the range of \(-1<\alpha<1\). Values of \( \beta \) for practical material combinations generally lie in the range of \( 0<\beta<\alpha/4 \). Interchanging materials 1 and 2 changes the sign of both \( \alpha \) and \( \beta \), and \( \alpha=\beta=0 \) is for the case of no material mismatch.

As discussed by Bogy using mentioned dimensionless parameters in Eqs. (1) and (2) for determination angles that singular stress occurs near the corner of dissimilar bonded planes for various combinations of material properties and joint geometry, the characteristic equation is obtained.
\[ D(\Theta_1, \Theta_2, \alpha, \beta, \lambda) = [4\lambda \sin(\Theta_1 - \Theta_2) + 4\lambda \sin(\Theta_1 + \Theta_2) + 4\lambda \sin(\Theta_1 + \Theta_2)] \cdot \alpha^2 + \\
2\lambda \cdot \alpha^2 \cdot \beta^2 + \\
4\lambda \cdot \alpha^2 \cdot \beta^2 \] (3)

Where, \( \Theta_1, \Theta_2 \) are the two singular angles and \( \lambda \) is an eigenvalue representing the stress singularity intensity.

Wu solved Eq. 3 for different \( E_1/E_2 \) ratios and presented the diagram for the \( 0 \leq \Theta_i \leq 180, (i = 1,2) \) case and \( \nu_1 = \nu_2 = 0.3 \). Figure 2 is divided into singular and non-singular zones by diagrams showing minimum singular angle.

\[ \theta_2 = 180 - \theta_1 \]

Figure 2. The range of minimum singular stress

According to figure 1, summation of \( \Theta_1, \Theta_2 \) becomes 180 degree. By plotting straight-line of \( \Theta_2 = 180 - \Theta_1 \) in figure 2, it is shown that \( 124 \leq \Theta_2 \leq 180 \) and \( 0 \leq \Theta_1 \leq 56 \) for elimination of singular angle. Because \( \Theta_1 \) is bigger than 90 degrees on point B in the second material, therefore the straight-side is not suitable for interface of bi-material bonded joint. More suitable profile for bi-material bonded joint interface for elimination of singular stress is shown in figure 3.

\[ \theta_2 = 180 - \theta_1 \]

Figure 3. Suitable profile for lake of stress singularity

In figure 3, \( \Theta_1 \) and \( \Theta_2 \) are angles between interface of first and second material, respectively. In this profile, angles that singular stress occurs should be calculated for \( \Theta_1 \) and \( \Theta_2 \). The range of \( \Theta_1, \Theta_2 \) was already computed. Singular angles around point C can be obtained by eigenvalues of \( \lambda \) through characteristic equation presented by Bogy.
$D(\theta_1, \theta_2, \alpha, \beta, \lambda) = [\lambda \sin^2 \theta_1 - (1 + \beta \sin^2 \lambda \theta_1) \times [\lambda \sin^2 \theta_1 - (1 - \beta \sin^2 \lambda \theta_1)] + (1 - \alpha) \sin \lambda (\pi - \theta_1)] + 2(1 - \beta \sin \lambda \theta_1 \sin \lambda \theta_1 + (1 - \alpha) \times \sin^2 \lambda (\pi - \theta_1))] = 0$  

Wu showed that there is always singular stress around point C, unless $\theta_3 = \theta_4 = 180$ and $\alpha = \beta$.

3. Particle swarm algorithm

In computer science, particle swarm optimization (PSO) is a computational method that optimizes a problem by iteratively trying to improve a candidate solution with regard to a given measure of quality. PSO optimizes a problem by having a population of candidate solutions, and moving these particles around in the search-space according to simple mathematical formulae over the particle's position and velocity. Each particle's movement is influenced by its local best known position and is also guided toward the best known positions in the search-space, which are updated as better positions are found by other particles. This is expected to move the swarm toward the best solutions. PSO is originally attributed to Kennedy and Shi [11, 12] as a stylized representation of the movement of organisms in a bird flock or fish school. The algorithm was simplified and it was observed to be performing optimization. PSO is a metaheuristic as it makes few or no assumptions about the problem being optimized and can search very large spaces of candidate solutions. However, metaheuristics such as PSO do not guarantee an optimal solution is ever found. More specifically, PSO does not use the gradient of the problem being optimized, which means PSO does not require that the optimization problem be differentiable as is required by classic optimization methods such as gradient descent and quasi-newton methods. PSO can therefore also be used on optimization problems that are partially irregular, noisy, change over time, etc.

In this paper, each particle is characterized by its position $X$ and velocity $V$. In the beginning, The PSO works by one group of random particles then algorithm searches in the solution space for finding optimized answer by updating position and velocity. In each step, each particle is updated by two best values. First value is the best answer that it has been achieved separately for each particle. It is named particle best (pbest); Second value is the best answer among entire particles which has been obtained separately, it is named global best (gbest). During the search process, each particle remembers its previous best solution, and if the current solution is better than the previous one, the position corresponding to current solution becomes the pbest; otherwise it remains unchanged. The best among all pbest solution is the gbest solution and the position corresponding to gbest value is the potential solutions of the problem.

New position and velocity of $i$ particle in $k$ step are updated by the following equations:

$$V_{i+1} = w^i V_{i+1} + c_1 \text{rand}_1 (P_{i+1} - X_i) + c_2 \text{rand}_2 (P_{i+1} - X_i)$$

$$X_{i+1} = X_i + V_{i+1}$$

Where $w^i$ represents inertia weight, $c_1$ and $c_2$ are learning factors which determine the relative influence of cognitive and social components, respectively. The value of $w^i$ is linearly decreased (adapted) with iteration in range of 0.9 to 0.4. rand1 and rand2 are independent random numbers uniformly distributed in the range of [0,1]. $c_1$ and $c_2$ values are numbers between 2 and 1.5. $V_{ik}$, $X_{ik}$ and $P_{ik}$ are the velocity, position and the personal best of $i$ particle for the $k$ iteration, respectively.

4. Geometrical modeling of interface joint

Presenting mathematical model of geometrical shape of machine or structure components plays an important role in shape optimizing. Generally, shape of joints should be reasonably modeled to achieve optimum shape easily by changing of model parameters. For this purpose, by using of smooth curve, like Bezier, interface of bi-material is modeled. This curve has few variables to change that causes great reductions in computational cost and also it can cover interface of shape very well. Consider a set of points (Figure 4),

$$P = (x_i, y_i) \quad i = 0, 1, 2, \ldots, n$$

![Figure 4. Bezier points](image-url)
x_i and y_i are control points. Consider polynomial of order n with n+1 points,

\[ p(u) = \sum_{i=0}^{n} \binom{n}{i} (1-u)^{n-i} u^i p_i \]  

(7)

Eq. 7 states two scalar equations for x_i and y_i in two dimensions. For n=3, two polynomial of order 3 is determined with 4 points p_0, p_1, p_2 and p_3,

\[ x(u) = (1-u)^3 x_0 + 3u(1-u)^2 x_1 + 3u^2 (1-u)x_2 + u^3 x_3 \]

\[ y(u) = (1-u)^3 y_0 + 3u(1-u)^2 y_1 + 3u^2 (1-u)y_2 + u^3 y_3 \]  

(8)

It is shown that if u=0, then x(0)=x_0, y(0)=y_0 and if u=1, x(1)=x_3, y(1)=y_3. It means that Bezier curve passes through the two end control points. If u varies between 0 and 1, curve is drawn from first point to fourth point. Curve shape can be transfigured by changing of controls points. In this paper, the Bezier curve with four control points is used.

5. Finite element model

The FE based computer code ANSYS is used for modeling and stress analyzing. The model is shown in figure 5 regarding symmetry to y axis. The PLANE 42 element has been used for meshing the model. It has 4 nodes and each node has 2 degrees of freedom.

![Figure 5. Finite element model](image)

6. Results and discussion

If the model is considered at angles where singular stress occurs, it will be found that stress rises increasingly. In addition, maximum von-mises stress at the interface corner is created within the material with higher modulus of elasticity (Figure 6).

![Figure 6. Stress contour in the model with stress singularity](image)
Now, if the model is considered at angles where there is no stress singularity, it will be found that stress contour is different from figure 6 of location and stress magnitude. Maximum Von-mises stress occurs at interface boundary of bi-material in the high elastic modulus material and occurs far from interface boundary of bi-material in low elastic modulus material.

![Figure 7. Stress contour without stress singularity](image)

Wu investigated various conditions and parameters including W (width), r (radius), R (radius) and γ (wedge angle) and determined that minimum stress concentration at boundary with circular shape was obtained in a way that $R = W / (2 \times \cos \gamma)$ where $\gamma=56^\circ$. It means that Wu’s circular shape is best shape for interface joint.

![Figure 8. Investigated shapes by Wu](image)

The obtained curve as bi-material interface differs from circular shape (Figure 9). This difference is due to that Wu introduced circular shape as the best shape among considered shapes but it is optimized in this research. Also, stress concentration coefficient resulted from this optimum curve is less than stress concentration in circular shape presented by Wu shown in figure 10. It is obvious that with constant Poisson’s ratio, by increasing $E_2/E_1$ ratio, stress concentration coefficient decreases.
Material properties influence optimal shape. As shown in Figure 11, convexity of first material with high elastic modulus within second material with low elastic modulus is increased with constant Poisson’s ratio and reduction of $\frac{E_2}{E_1}$ ratio.

Figure 9. Comparison between optimal curve and circular shape

Figure 10. Stress concentration comparison between optimal curve and circular shape at various $E_2/E_1$ ratio

Figure 11. Effect of material properties on optimal boundary between two materials
7. Conclusions

In this paper, shape optimization of bi-martial bonded joint is investigated based on particle swarm algorithm. ANSYS parametric Design Language (APDL) is implemented in order to minimize stress concentration without stress singularity. The results indicate that with constant Poisson’s ratio, stress concentration decreases by increasing the ratio of elastic modulus. Furthermore, Numerical examples display significant decrease in stress concentration in optimum shape compared to circular shapes.

References


